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Filter for strangeness in J^{PC} exotic four-quark states

P.R. Page[a]

Theoretical Division, MS B283, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

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Abstract. Symmetrization selection rules for the decay of four–quark states to two $J = 0$ mesons are analysed in a non – field theoretic context with isospin symmetry. The OZI allowed decay of an isoscalar $J^{PC} = \{1, 3, \ldots\}^{-+}$ exotic state to $\eta^{'}\eta$ or $f_0^{'}f_0$ is only allowed for four–quark components of the state containing one $s\bar{s}$ pair, providing a filter for strangeness content in these states. Decays of four–quark a_0 states are narrower than otherwise expected. If the experimentally observed 1^{-+} enhancement in $\eta\pi$ is resonant, it is qualitatively in agreement with being a four–quark state.

1 Introduction

Ever since the original work in the MIT bag model, it has been recognized that multiquark states containing strange quarks can often have *lower* energies than those with only the equivalent light (up or down) quarks [1], leading to the prediction of the stability of strangelets. For four–quark states, built from two quarks (q) and two antiquarks ($\bar{q}$), the same conclusion was reached in potential models [2].

Hybrid mesons ($q\bar{q}$ with a gluonic excitation), glueballs (gluonic excitation without $q\bar{q}$) and four–quark states have certain vanishing decays due to symmetrization selection rules [3]. In this Paper symmetrization selection rule II [3], i.e. the case of isospin symmetry, is exhaustively analysed for the decay of four–quark states to two internal angular momentum $J = 0$ hybrid or conventional mesons, expanding and superseding the earlier analysis [3]. The decay of (hybrid) mesons and glueballs to two $J = 0$ (hybrid) mesons were considered before [3]. The possibility of six–quark or higher multiquark states is not considered. We say that the combination of J, P (parity) and C (charge conjugation) is "J^{PC} exotic" if conventional mesons cannot have these J^{PC}. It is shown that certain decays signal the presence of strangeness in decaying J^{PC} exotic four–quark states, providing an experimental tool to verify the claimed presence of strangeness in these states. Decays also allow us to distinguish between the hybrid, glueball or four–quark character of a decaying J^{PC} exotic state. There are also implications for non–exotic four–quark states.

2 Formalism

We first consider states built only from isospin $\frac{1}{2}$ quarks, i.e. u and d quarks. For four–quark states we are free to choose any basis to construct their flavour parts, called the "flavour states". Labelling the quarks and antiquarks in the four–quark state as $q_1\bar{q}_2q_3\bar{q}_4$, and grouping $q_1\bar{q}_2$ and $q_3\bar{q}_4$ (denoted by X and Y) together as a choice of basis, the four–quark flavour states are

$$|I_A I_A^z I_X I_Y\rangle \equiv \sum_{I_X^z I_Y^z} \langle I_A I_A^z | I_X I_X^z I_Y I_Y^z\rangle |X\rangle\ |Y\rangle \qquad (1)$$

Here $|X\rangle$ and $|Y\rangle$ have definite isospin (projection) I_X (I_X^z) and I_Y (I_Y^z) respectively. The isospin (projection) of $|X\rangle$ and $|Y\rangle$ was combined by use of Clebsch–Gordon coefficients to obtain the total isospin (projection) I_A (I_A^z) of the four–quark state A, by summing over isospin projections[1]. States can be verified to satisfy the orthonormality condition $\langle I_A I_A^z I_X I_Y | I_A^{'} I_A^{z'} I_X^{'} I_Y^{'}\rangle = \delta_{I_A I_A^{'}} \delta_{I_A^z I_A^{z'}} \delta_{I_X I_X^{'}} \delta_{I_Y I_Y^{'}}$.

In this Paper we consider four–quark states with integral isospin. When $I_A = 0$, the physical state is a linear combination of components with flavour states $|0\ 0\ 0\ 0\rangle$ and $|0\ 0\ 1\ 1\rangle$. For $I_A = 2$, the physical state has flavour state $|2\ I_A^z\ 1\ 1\rangle$. For $I_A = 1$, we define new flavour states $|1\ I_A^z \pm\rangle \equiv \frac{1}{\sqrt{2}}(|1\ I_A^z 1\ 0\rangle \pm |1\ I_A^z 0\ 1\rangle)$. When $I_A = 1$, the physical state is a linear combination of components with

[a] e-mail: prp@lanl.gov

[1] Because X and Y are merely labels, the flavour states will be constructed to be representations of the label group, i.e. either symmetric or antisymmetric under $X \leftrightarrow Y$ exchange. Models where the dynamics are truncated [15] in such a way that $q_1\bar{q}_2$ occur in one meson, and $q_3\bar{q}_4$ in another, i.e. where four–quark states are viewed as molecules of mesons, are *not* included in our discussion. This is because, e.g. for an $\eta\pi$ molecule, one can define q_1 and $\bar{q}_2$ to be in η. Label symmetry requires that q_1 and $\bar{q}_2$ can also be in π. But this is impossible by assumption. It should be noted that there is in general nothing special about $q_1\bar{q}_2$ as opposed to $q_3\bar{q}_4$, so that $X \leftrightarrow Y$ exchange is allowed

Table 1. Explicit neutral four–quark flavour states

Isospin 2 four–quark:		$\|000s\bar{s}\rangle$	$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})s\bar{s}$
$\|2011\rangle$	$\frac{1}{\sqrt{6}}(-u\bar{d}d\bar{u}-d\bar{u}u\bar{d}$ $+u\bar{u}u\bar{u}-u\bar{u}d\bar{d}-d\bar{d}u\bar{u}+d\bar{d}d\bar{d})$	Isospin 1 four–quark:	
Isospin 0 four–quark:		$\|1011\rangle$	$\frac{1}{\sqrt{2}}(d\bar{u}u\bar{d}-u\bar{d}d\bar{u})$
$\|0000\rangle$	$\frac{1}{2}(u\bar{u}u\bar{u}+u\bar{u}d\bar{d}+d\bar{d}u\bar{u}+d\bar{d}d\bar{d})$	$\|10+\rangle$	$\frac{1}{\sqrt{2}}(u\bar{u}u\bar{u}-d\bar{d}d\bar{d})$
$\|0011\rangle$	$-\frac{1}{\sqrt{3}}(u\bar{d}d\bar{u}+d\bar{u}u\bar{d}$	$\|10-\rangle$	$\frac{1}{\sqrt{2}}(u\bar{u}d\bar{d}-d\bar{d}u\bar{u})$
	$+\frac{1}{2}(u\bar{u}u\bar{u}-u\bar{u}d\bar{d}-d\bar{d}u\bar{u}+d\bar{d}d\bar{d}))$	$\|101s\bar{s}\rangle$	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})s\bar{s}$

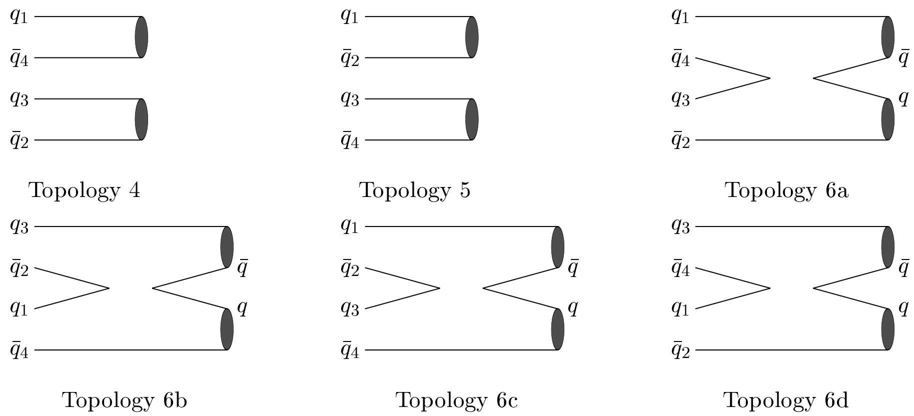

Fig. 1. Connected topologies

flavour states $|1 I_A^z 1 1\rangle$, $|1 I_A^z +\rangle$ and $|1 I_A^z -\rangle$. The presence of $s\bar{s}$ pairs is now explored. By convention, we choose a single strange pair to correspond to labels $q_3 = s$ and $\bar{q}_4 = \bar{s}$, so that 1 and 2 still labels u and d quarks. The four–quark flavour state is $|I_A I_A^z I_X s\bar{s}\rangle \equiv |X\rangle\ s\bar{s}$, either isovector or isoscalar. Another possibility is $|00c\bar{c}s\bar{s}\rangle \equiv c\bar{c}s\bar{s}$. For two strange pairs, the flavour state is $|00s\bar{s}s\bar{s}\rangle \equiv s\bar{s}s\bar{s}$. Other states are obtained by freely interchanging strange, charm and bottom quarks. Explicit forms for some of the neutral flavour states are given in Table 1.

We shall be interested in decay and production $A \leftrightarrow BC$ processes in the rest frame of A. For simplicity we shall usually refer to the decay process $A \rightarrow BC$, but the statements will be equally valid for the production process $A \leftarrow BC$. The decay of an isospin I_A four–quark state to two states with integral isospins I_B and I_C is considered [4]. We shall restrict B and C to $J = 0$ states with quark–antiquark content. The states can be thought of as having arbitrary gluonic excitation, i.e. as hybrid or conventional mesons, and as being radial excitations or ground states. B and C have $J^P = 0^-$ or 0^+. If charge conjugation is a good quantum number, $J^{PC} = 0^{-+}, 0^{+-}, 0^{++}$ or 0^{--} are allowed. Since 0^{-+} and 0^{++} ground state conventional meson states B and C are most likely to be allowed by phase space, they are used in the examples.

Assume that states B and C are identical in all respects except, in principle, their flavour and their equal but opposite momenta $\mathbf{p}$ and $-\mathbf{p}$ (in the rest frame of A). Hence B and C have the same parity, charge conjugation, radial and gluonic excitation, as well as the same internal structure. However, they are not required to have the same energies or masses [3]. One candidate example is η and π. Although the previous condition on B and C is not expected to be exactly realized in nature unless they are identical particles, it can be realized in theoretical calculations.

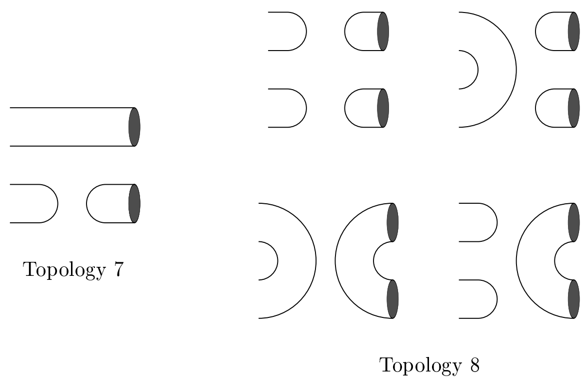

Fig. 2. Disconnected topologies

The interactions come from the strong interactions described by QCD. The quarks and antiquarks in A travel in all possible complicated paths going forward and backward in time and emitting and absorbing gluons until they emerge in B and C. Figures 1 and 2 indicate the possible ways ("topologies") the quarks and antiquarks in the four–quark state on the left hand side can rearrange themselves into the quarks and antiquarks in the two (hybrid) mesons (the blobs) on the right hand side. Gluons and quark loops are not indicated. We shall assume that the topologies indicated are all the possibilities allowed by QCD. This is certainly the case in perturbative QCD and also emerges after gluon field integration in the path integral formalism [5]. We shall assume a framework where the initial state A and final states B and C with their corresponding wave functions are "attached" to each topology. This framework is often used in model calculations, but, as we shall see later, is not a field theoretic treatment. We also assume that the experimentally observed (resonant) states A, B and C admit a Fock state expansion in terms of states, each of which has certain "valence" quarks and antiquarks with arbitrary gluonic content. Particularly, this Paper specializes to a four–quark Fock state A and (hybrid) meson Fock states B and C. It is not assumed that A is a pure four–quark state: we specialize to the four–quark Fock state because the hybrid meson and glueball Fock states have already been analysed before [3].

In Fig. 1, for each quark (or antiquark) in A that ends up in B, there is also the possibility that it would end up in C. Hence a given topology in Fig. 1, e.g. 6a, stands for *two* topologically distinct parts. Furthermore, each of topologies 4–6 is separately distinct. They are labelled analogous to earlier conventions [3]. Each topologically distinct part (decay amplitude) in Figs. 1 and 2 is a product of a flavour overlap $\mathcal{F}$ and a "remaining" overlap. We shall be interested in the exchange properties of $\mathcal{F}$ when the labels that specify the flavour of the states B and C are formally exchanged, denoted by $B \leftrightarrow C$. In cases where $\mathcal{F}$ is non–zero and transforms into itself, which will yield vanishing decays by symmetrization selection rules, define $\mathcal{F}_{B\leftrightarrow C} \equiv f\mathcal{F}$. If $\mathcal{F}$ has no simple transformation properties under $B \leftrightarrow C$ exchange, we cannot obtain symmetrization selection rules.

It is possible to omit the following proof of the results of this Paper and continue directly to the statement of the results, which can be found in the next section.

The flavour state of a $q\bar{q}$ pair is

$$|H\rangle = \sum_{h\bar{h}} H_{h\bar{h}}|h\rangle|\bar{h}\rangle \qquad \text{where}$$

$$H_{h\bar{h}} = \langle I_H I_H^z|\frac{1}{2}h\frac{1}{2}-\bar{h}\rangle(-1)^{\frac{1}{2}-\bar{h}} \tag{2}$$

where $|\frac{1}{2}\rangle = u$, $|-\frac{1}{2}\rangle = d$, $|\bar{\frac{1}{2}}\rangle = \bar{u}$ and $|-\bar{\frac{1}{2}}\rangle = \bar{d}$. This just yields the usual $I = 1$ flavour $-u\bar{d}$, $\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$, $d\bar{u}$ for $I^z = 1, 0, -1$ and $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$ for $I = 0$. The advantage of this way of identifying flavour is that any pair creation or annihilation that takes place will do so with $I = 0$ pairs $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) = \frac{1}{\sqrt{2}}\sum_{h\bar{h}}\delta_{h\bar{h}}|h\rangle|\bar{h}\rangle$ formed out of gluons, making the operator trivial.

In order to illuminate the method, we discuss the case where only u and d quarks participate in the decay. The presence of strange quarks only simplifies the overlap. From (1) and (2), the flavour overlap $\mathcal{F}$ equals

$$\begin{aligned}\sum_{a_1\,\bar{a}_2\,a_3\,\bar{a}_4\,b\,\bar{b}\,c\,\bar{c}\,I_X^z\,I_Y^z} &\langle I_A I_A^z|I_X I_X^z I_Y I_Y^z\rangle \\ &\times\langle I_X I_X^z|\frac{1}{2}a_1\frac{1}{2}-\bar{a}_2\rangle\,(-1)^{\frac{1}{2}-\bar{a}_2} \\ &\times\langle I_Y I_Y^z|\frac{1}{2}a_3\frac{1}{2}-\bar{a}_4\rangle\,(-1)^{\frac{1}{2}-\bar{a}_4} \\ &\times\langle I_B I_B^z|\frac{1}{2}b\frac{1}{2}-\bar{b}\rangle\,(-1)^{\frac{1}{2}-\bar{b}} \\ &\times\langle I_C I_C^z|\frac{1}{2}c\frac{1}{2}-\bar{c}\rangle\,(-1)^{\frac{1}{2}-\bar{c}}\mathrm{KD}\end{aligned} \tag{3}$$

Table 2. Behaviour of the (non–vanishing) flavour overlap $\mathcal{F}$ for the decay of the indicated four–quark flavour state A to mesons B and C under $B \leftrightarrow C$ flavour label exchange, i.e. $\mathcal{F}_{B\leftrightarrow C} = f\mathcal{F}$, in the topology under consideration. $i \equiv (-1)^{I_A+I_B+I_C}$. The symbol $\ni$ indicates that $\mathcal{F}_{B\leftrightarrow C} \neq f\mathcal{F}$, i.e. that there are no symmetrization selection rules. If a state is not indicated for a given topology it means that $\mathcal{F}$ vanishes, so that the decay is not allowed. When decay is not allowed by isospin conservation, $\mathcal{F} = 0$ as expected. This happens when $\mathbf{I_A} \neq \mathbf{I_B} + \mathbf{I_C}$ or $I_A^z \neq I_B^z + I_C^z$, or when $I_A = I_B = I_C = 1$ and $I_A^z = I_B^z = I_C^z = 0$

Isospin 0 four–quark			Isospin 1 four–quark			Isospin 2 four–quark		
Top.	State	f	Top.	State	f	Top.	State	f
4	$\|0000\rangle$	i	4	$\|1I_A^z11\rangle$ [a]	$-i$	4	$\|2I_A^z11\rangle$	i
	$\|0011\rangle$	i		$\|1I_A^z+\rangle$ [a]	i	5	$\|2I_A^z11\rangle$	i
	$\|00s\bar{s}s\bar{s}\rangle$ [c]	i		$\|1I_A^z-\rangle$ [b]	i			
5	$\|0000\rangle$ [c]	i	5	$\|1I_A^z11\rangle$ [b]	i			
	$\|0011\rangle$ [b]	i		$\|1I_A^z+\rangle$ [a]	i			
	$\|000s\bar{s}\rangle$ [c]	$\ni$		$\|1I_A^z-\rangle$ [a]	$-i$			
	$\|00c\bar{c}s\bar{s}\rangle$ [c]	$\ni$		$\|1I_A^z1s\bar{s}\rangle$ [a]	$\ni$			
	$\|00s\bar{s}s\bar{s}\rangle$ [c]	i	6a,b	$\|1I_A^z+\rangle$	i			
6a,b	$\|0000\rangle$	i		$\|1I_A^z-\rangle$	i			
	$\|000s\bar{s}\rangle$ [d]	i		$\|1I_A^z1s\bar{s}\rangle$ [e]	i			
	$\|00c\bar{c}s\bar{s}\rangle$ [c]	i	6c,d	$\|1I_A^z11\rangle$	i			
	$\|00s\bar{s}s\bar{s}\rangle$ [c]	i		$\|1I_A^z+\rangle$	i			
6c,d	$\|0000\rangle$	i						
	$\|0011\rangle$	i						
	$\|00s\bar{s}s\bar{s}\rangle$ [c]	i						

[a] $\mathcal{F} \neq 0$ only if $I_B \neq I_C$
[b] $\mathcal{F} \neq 0$ only if $I_B = I_C = 1$
[c] $\mathcal{F} \neq 0$ only if $I_B = I_C = 0$
[d] In topology 6b $\mathcal{F} \neq 0$ only if $I_B = I_C = 0$
[e] $\mathcal{F} \neq 0$ only in topology 6a

The states $|B\rangle$ and $|C\rangle$ have isospin (projection) I_B (I_B^z) and I_C (I_C^z) respectively. "KD" is a set of Kronecker delta functions that specifies how the quark lines connect in the decay topology. Specialize to topology 6a as an example. From Fig. 1 "KD" is $\delta_{a_1b}\delta_{\bar{a}_2\bar{c}}\delta_{a_3\bar{a}_4}\delta_{\bar{b}c}$. If one formally interchanges all labels B and C in (3), it can be verified that $\mathcal{F} \rightarrow (-1)^{I_X+I_B+I_C}\mathcal{F}$. Since the overlap is non–zero only when $I_Y = 0$ (due to the $q_3\bar{q}_4$ pair annihilating), it follows by conservation of isospin that $I_A = I_X$, so that $\mathcal{F} \rightarrow i\mathcal{F}$, where $i \equiv (-1)^{I_A+I_B+I_C}$. Thus $f = i$. This, as well as the fact that the overlap vanishes when $I_Y = 1$, are indicated in Table 2.

Let C^0 be the charge conjugation of a neutral state. For charged states (for which charge conjugation is not a good quantum number), we note that at least one of the states in the isomultiplet it belongs to has a well–defined charge conjugation, denoted by C^0. G–parity conservation $G_A = G_B G_C$ in isospin symmetric QCD and the relation $G = (-1)^I C^0$ imply that $C_A^0 = i$, as was noted in Sect. 2.2 of [3], using that $C_B^0 = C_C^0$ as assumed earlier.

It was shown in (3) of [3] that the decay vanishes due to symmetrization selection rules if the parity $P_A = -f$. The argument will not be repeated here. If $f = i$, then

$P_A = -f = -i = -C_A^0$, i.e. a neutral state A is CP odd. Since states B and C both have $J = 0$, it follows by conservation of angular momentum that an L–wave decay would necessitate $J_A = L$. Using that $P_B = P_C$ by assumption, conservation of parity in QCD necessitates $P_A = (-1)^L$. Hence neutral states A have $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$, which are all exotic J^{PC}. A charged state A (with no charge conjugation) should have a neutral isopartner with the foregoing J^{PC}. If $f = -i$, the same reasoning shows that neutral states A with non–exotic $J^{PC} = 0^{++}, 1^{--}, 2^{++}, 3^{--}, \ldots$, and charged states with a neutral isopartner with these J^{PC}, have vanishing decay.

3 Results

Topologies 4–6 in Fig. 1 are called "connected" and are allowed by the Okubo–Zweig–Iizuka (OZI) rule [6], while topologies 7–8 in Fig. 2 are "disconnected" and suppressed by the OZI rule. Topologies 4 and 5 are called "fall apart" topologies because the four–quark state simply falls apart into two (hybrid) mesons. For topology 7 we do not find vanishing decays coming from symmetrization selection rules. Topology 8 is discussed further below. The *results* of our analysis for topologies 4–6, which are expected to be dominant by the OZI rule, are summarized in Table 2. Isospin 0 four–quark states can contain components with flavour states $|0000\rangle$ or $|0011\rangle$ (u, d quarks only), $|000s\bar{s}\rangle$ or $|00c\bar{c}s\bar{s}\rangle$ (one $s\bar{s}$ pair), or $|00s\bar{s}s\bar{s}\rangle$. Isospin 1 states have flavour states $|1I_A^z11\rangle$, $|1I_A^z+\rangle$ or $|1I_A^z-\rangle$ (u, d quarks only) or $|1I_A^z1s\bar{s}\rangle$ (one $s\bar{s}$ pair). Isospin 2 states have the flavour state $|2I_A^z11\rangle$ with u, d quarks only. Other states are obtained by freely interchanging strange, charm and bottom quarks, and are not indicated. Only those flavour states which are allowed to decay in a given topology are indicated. The symbol $\ni$ denotes that the decay is not vanishing due to symmetrization selection rules. *In the topology in Fig. 1 under consideration, an entry i in Table 2 indicates that the decay of the component of the physical state with the corresponding four–quark flavour state vanishes by symmetrization selection rules for $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$ four–quark states. Ditto for an entry $-i$, except that the four–quark state has $J^{PC} = 0^{++}, 1^{--}, 2^{++}, 3^{--}, \ldots$.* It immediately becomes clear that the decay of the four–quark states with the J^{PC} just mentioned is less than what one would naïvely expect, making them more stable.

To make the use of Table 2 clear, we consider the example of the decay of an isovector 1^{-+} state to $\eta\pi$ in topologies 4–6. The physical 1^{-+} state is a linear combination of components with flavour states $|1I_A^z11\rangle$, $|1I_A^z+\rangle$, $|1I_A^z-\rangle$ and $|1I_A^z1s\bar{s}\rangle$. Referring to Table 2, the component with $|1I_A^z11\rangle$ decays in topology 4 only, $|1I_A^z-\rangle$ in topology 5 only and $|1I_A^z1s\bar{s}\rangle$ in topology 5 only. The decay of the component $|1I_A^z+\rangle$ vanishes.

The implications of Table 2 for the two J^{PC} sequences are now analysed. The validity of the discussion should be viewed within the context of the restrictions on the final states B and C discussed earlier.

3.1 Decay of $J^{PC} = 0^{+-},\ 1^{-+},\ 2^{+-},\ 3^{-+}, \ldots$ exotic four–quark states to two $J = 0$ (hybrid) mesons

1. If $I_A = 2$ or $I_A = I_B = I_C = 1$, contributions from *all* four–quark topologies vanish. They also vanish for all hybrid meson and glueball topologies [3]. This partially follows by noting that $I_B = I_C = 1$, and that topology 7 involves pair creation which cannot occur in isospin 1; and that $I_A \neq 0$, and that topology 8 completely annihilates the four–quark state which cannot occur when $I_A \neq 0$. If $I_A = 0$ and $I_B = I_C = 1$, contributions from all connected four–quark topologies vanish. They also vanish for the connected hybrid meson topology [3].
2. Contributions from all "non – fall apart" connected topologies 6 vanish.
3. If $I_A = 0$ and $I_B = I_C = 0$, and the decay is non–vanishing, this comes from either a four–quark component with a single $s\bar{s}$ pair which decays via "fall apart" connected topology 5 or from disconnected topologies. Also note that the decay cannot come from connected hybrid meson decay [3]. Assuming the OZI rule that disconnected topologies are suppressed, one discovers that a non–vanishing decay only comes from a single $s\bar{s}$ four–quark component. This isolates the presence of an $s\bar{s}$ component in the state, i.e. acts like a *strangeness filter*. Its presence is expected, as $u\bar{u}$ and $d\bar{d}$ components of a four–quark state can in perturbation theory be expected to mix substantially via single gluon exchange with $s\bar{s}$ [7], with flavour mixing of this kind $\lesssim 10\%$ in a model calculation [2].
4. If $I_A = 1$ and $I_B \neq I_C$, connected decay does not come from the $|1I_A^z+\rangle$ component.

Examples: There are no examples involving $\pi\pi$ and a_0a_0 final states that are not forbidden by well–known selection rules of QCD, e.g. G–parity or CP conservation, or generalized Bose symmetry (Bose symmetry of identical final states extended to final states in the same isomultiplet). Hence there is no new selection rules arising from item 1. From the last two items we obtain the following examples (where the f_0, f_0' and a_0 final states are taken to be mesons):

Item 3: Isoscalar $1^{-+},\ 3^{-+}, \ldots \to \eta'\eta,\ f_0'f_0$ indicates a four–quark component with a single $s\bar{s}$ in the initial state.

Item 4: Isovector $1^{-+},\ 3^{-+}, \ldots \to \eta\pi,\ \eta'\pi,\ f_0a_0,\ f_0'a_0$ connected decay does not come from a $|1I_A^z+\rangle$ component in the initial state.

3.2 Decay of $J^{PC} = 0^{++},\ 1^{--},\ 2^{++},\ 3^{--}, \ldots$ non–exotic four–quark states to two $J = 0$ (hybrid) mesons

In the cases that $I_A = 1$ and $I_B \neq I_C$ some contributions vanish, making the states narrower than otherwise expected.

Examples: Isovector $0^{++},\ 2^{++}, \ldots \to \eta\pi,\ \eta'\pi,\ f_0a_0,\ f_0'a_0$ is narrower than otherwise expected.

The decays can only be found to vanish by symmetrization selection rules if the quark structure of the decay is analysed. Models which only analyse decay at the hadronic level, do not incorporate the selection rule: The decay of four–quark $a_0(980) \to \eta\pi$ was recently modelled at the hadronic level [8].

4 Remarks

The previous section constitutes the results of this Paper. A few final remarks are in order.

If one does not assume isospin symmetry, i.e. considers interactions described by both QCD and QED, one can apply symmetrization selection rule I of [3]: the case without isospin symmetry [9].

Consider topology 8 where a "half–doughnut" or two "raindrops" is created from the vacuum after the four–quark state has annihilated. There are similar topologies for an initial meson or glueball [3]. These topologies can be analysed without the need for isospin symmetry. Symmetrization selection rules for the "half–doughnut" can be shown to apply only for decays already known to vanish by CP conservation or Bose symmetry [3]. From symmetrization selection rule III of [3], decay of $J^{PC} = 1^{-+}, 3^{-+}, \ldots$ four–quark states in "raindrop" topologies vanishes in those cases where the $B \leftrightarrow C$ exchanged diagram is topologically distinct from the original diagram.

It needs to be emphasized that this Paper analyses the flavour structure of various decay topologies in a generic way, which should subsume the treatments of numerous models of QCD, and hence serves as a check on model calculations. However, it is *not* a field theoretic treatment, and hence cannot be regarded as giving predictions of QCD as a field theory. This becomes evident when one studies the following condition for the validity of our conclusions. We assume that states B and C are identical in all respects except, in principle, their flavour. Although this requirement is needed here, it is not necessary in field theory, as a recent analysis demonstrates [5]: The requirement is not needed for at least on–shell η and π states B and C in a certain energy range and for certain quark masses. Our analysis in terms of topologies has application to the analysis of Green's functions with three currents, for example diagrams in QCD sum rules, or topologies obtained via gluon field integration in the path integral formalism [5]. However, when QCD as a field theory is considered, the difference with our analysis is that the Green's function is not the same as the decay amplitude of the particles A, B and C.

The final states B and C are taken to be (hybrid) meson Fock states in our analysis. Our conclusions would not be applicable to physical states B or C which have significant non–meson Fock state components. The η' is thought to acquire a glueball Fock state via the $U(1)$ anomaly. However, QCD sum rule calculations indicate that 1^{-+} hybrid meson decay to $\eta'\pi$ proceeding through the glueball component of the η' only give a width of $3-5$ MeV [10]. This corresponds to the finding that there is no significant experimental evidence for a glueball component in the η', when $J/\psi \to 0^-1^-$ hadronic decay and η' two–photon and radiative decay data are considered together [11]. For the f_0, f_0' and a_0 states, especially the low–lying $\sigma(400-1200)$, $f_0(980)$ and $a_0(980)$ [12], which are most likely to be allowed by phase space, the non–meson Fock state composition is not currently well known.

A candidate isovector state $\hat{\rho}(1405)$ with width 333±50 MeV, decaying to $\eta\pi$, and possibly to $\eta'\pi$, has been reported [12]. It is interesting to note that a quark model calculation finds the lightest 1^{-+} four–quark state at 1418 MeV, although it is an isoscalar with flavour state $|000s\bar{s}\rangle$ [2]. The isovector state is heavier [13]. If the $\hat{\rho}(1405)$ is resonant and has a substantial branching ratio of $\eta\pi$, this decay mode may discriminate against the hybrid interpretation of the state. This is because a hybrid meson only decays via a (presumably suppressed) OZI forbidden topology [3,5]. The symmetrization selection rules have interesting applications even if the initial state A is a mixture of various Fock states[2]. For example, $\hat{\rho}(1405)$ can not have a glueball Fock state, because it is isovector, and cannot have a meson Fock state, because it is J^{PC} exotic. If $\hat{\rho}(1405)$ is a mixture of hybrid meson and four–quark Fock states [14], and we restrict to OZI allowed decay topologies, the symmetrization selection rules yield that decay to $\eta\pi$ and $\eta'\pi$ only comes from the four–quark Fock state. Particularly, we predict that the decay arises from *only* certain four–quark components, so that the detection of substantial branching ratios in $\eta\pi$ or $\eta'\pi$ signals the presence of such components.

If the isoscalar partner of $\hat{\rho}(1405)$ is a mixture of hybrid meson, four–quark and glueball Fock states, OZI allowed decay only comes from hybrid meson and four–quark Fock states, since a glueball Fock state yields OZI forbidden decays. Symmetrization selection rules yield that OZI allowed decay to $\eta'\eta$ only comes from the four–quark Fock state, specifically from a component with a single $s\bar{s}$ in the initial state.

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References

1. R.L. Jaffe, Phys. Rev. **D15**, 267 (1977); ibid. 281; **D17**, 1444 (1978); Chan H.-M., H. Høgaasen, Phys. Lett. **B72**, 400 (1978)
2. C. Semay, B. Silvestre–Brac, Phys. Rev. **D51**, 1258 (1995)
3. P.R. Page, Phys. Lett. **B401**, 313 (1997), and references therein
4. Final states with flavour $c\bar{s}$, $s\bar{c}$, $b\bar{s}$, $s\bar{b}$, $c\bar{b}$, $b\bar{c}$ are not considered
5. P.R. Page, Phys. Rev. **D64** (2001) 056009

[2] It is not admissible to consider a process where a Fock state 1 mixes with an intermediate Fock state 2 which then decays. This amounts to double counting, because the physical state is already considered as a mixture of Fock states 1 and 2

6. S. Okubo, Phys. Lett. **B5**, 165 (1963); G. Zweig, CERN Report No. 8419/TH412 (1964); I. Iizuka, Prog. Theor. Phys. **38**, 21 (1966)
7. F.E. Close, H.J. Lipkin, Phys. Lett. **B196**, 245 (1987)
8. D. Black, A.H. Fariborz, J. Schechter, Phys. Rev. **D61** (1999) 074001
9. It is possible to show that the behaviour of $\mathcal{F}$ under $B \leftrightarrow C$ exchange remains the same as in Table 2 even without assuming isospin symmetry, when a neutral four-quark state decays to two neutral states, i.e. when $I_A^z = I_B^z = I_C^z = 0$ in Table 2. For the decay of these states in topologies 6 it is seen that $f = i$ in Table 2, so that item 2 remains valid without assuming isospin symmetry. This can independently be verified from symmetrization selection rule I
10. S. Narison, "QCD spectral sum rules", Lecture Notes in Phys. Vol. 26 (1989), p. 374; J.I. Latorre et al., Z. Phys. **C34**, 347 (1987)
11. J.L. Rosner, Proc. of 2^{nd} Int. Conf. on Hadron Spectroscopy (Hadron 87) (April 1987, Tsukuba, Japan), p. 395, eds. Y. Oyanagi et al.
12. Particle Data Group (C. Caso et al.), Eur. Phys. J. **C3**, 1 (1998)
13. C. Semay, private communication
14. N.N. Achasov, G.N. Shestakov, Phys. Rev. **D63** (2000) 014017
15. R. Zhang, Y.-B. Ding, X.-Q. Li, P.R. Page, in preparation